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과목코드	34	논술형 답지	과목명	미적분학 II
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DH EDU

학번 : ____________ **이름 :** ____________

총 문항 수

구분	선다형	논술형	기타
문항 수	24	0	0
배점	100	0	0
점수 합계	100		

1. Find $f(5)$ when $f'(x)=e^{2x}$ and $f(0)=2$.

① $\dfrac{e^{10}}{2}$

② $\dfrac{e^{10}+1}{2}$

③ $\dfrac{e^{10}}{2}+1$

④ $\dfrac{e^{10}+3}{2}$

⑤ $\dfrac{e^{10}}{2}+2$

2. Find $f(1)$ when $f'(x)=\dfrac{1}{\sqrt{4-x^2}}$ and $f(0)=0$

① 0

② $\dfrac{\pi}{6}$

③ $\dfrac{\pi}{4}$

④ $\dfrac{\pi}{3}$

⑤ $\dfrac{\pi}{2}$

3. Find $\displaystyle\int(1+2x)^3dx$.

① $\dfrac{1}{2}(1+2x)^4+C$

② $\dfrac{1}{4}(1+2x)^4+C$

③ $\dfrac{1}{8}(1+2x)^4+C$

④ $\dfrac{1}{10}(1+2x)^4+C$

⑤ $\dfrac{1}{16}(1+2x)^4+C$

4. Find $\displaystyle\int 2^{3x+4}dx$.

① $\dfrac{1}{6\ln 2}2^{3x+4}+C$

② $\dfrac{1}{3\ln 2}2^{3x+4}+C$

③ $\dfrac{1}{2\ln 2}2^{3x+4}+C$

④ $\dfrac{2}{3\ln 2}2^{3x+4}+C$

⑤ $\dfrac{5}{6\ln 2}2^{3x+4}+C$

5. Find the expression for the area of the region as a limit, that lies under the graph of $f(x) = x^3$ between $x=1$ and $x=2$ using the left end point.

① $\lim_{n\to\infty}\sum_{k=0}^{n}\frac{1}{n}\cdot(1+\frac{k+1}{n})^3$

② $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\cdot(1+\frac{k}{n})^3$

③ $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\cdot(1+\frac{2k}{n})^3$

④ $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\cdot(1+\frac{2k-2}{n})^3$

⑤ $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\cdot(1+\frac{k-1}{n})^3$

6. Find $\int_{\frac{\pi}{2}}^{0}\cos\theta\, d\theta$.

① -1

② $-\frac{1}{2}$

③ 0

④ $\frac{1}{2}$

⑤ 1

7. Find $\int_{e}^{e^4}\frac{\sqrt{\ln x}}{x}dx$.

① 2

② $\frac{10}{3}$

③ $\frac{14}{3}$

④ 6

⑤ 9

8. Find $\int_{0}^{1}x\sqrt{1-x^4}\,dx$.

① $\frac{\pi}{16}$

② $\frac{\pi}{8}$

③ $\frac{\pi}{4}$

④ $\frac{\pi}{2}$

⑤ π

9. Find $F'(1)$ when $F(x)=\int_{x}^{x^2} e^{t^2}dt$.

① e^{-1}
② 1
③ e
④ e^2
⑤ e^4

10. Which of the following has the same value with $\int_0^1 e^{\sqrt{x}}dx$?

Ⓐ $\int_0^{\frac{\pi}{2}} e^{\sin x}\sin 2x\,dx$
Ⓑ $\int_1^{e} 2\ln x\,dx$
Ⓒ $\int_0^{1} xe^{x}\,dx$

① Ⓐ only
② Ⓐ and Ⓑ only
③ Ⓑ and Ⓒ only
④ Ⓐ and Ⓒ only
⑤ Ⓐ, Ⓑ, and Ⓒ

11. The linear density in a rod 15m long is $\frac{20}{\sqrt{x+1}}$kg/m, where x is measured in meters from one end of the rod. Find the average density (kilograms per meter) of the rod.

① 2
② 4
③ 6
④ 8
⑤ 10

12. Find the area of the region encclosed by the graphs of $y=3^x, y=2^x$, $x=-1$, and $x=1$.

① $\frac{1}{\ln(3)}-\frac{1}{\ln(2)}$
② $\frac{4}{3\ln(3)}-\frac{1}{\ln(2)}$
③ $\frac{1}{\ln(3)}-\frac{1}{2\ln(2)}$
④ $\frac{4}{3\ln(3)}-\frac{2}{\ln(2)}$
⑤ $\frac{4}{3\ln(3)}-\frac{1}{2\ln(2)}$

13. Water flows into the reservoir at a rate of $f(t) = 2700 - 300t^2$ liters per hour, where t is measured in hours ($0 \le t \le 3$). Find the amount (liters) of water that flows from the tank during $\underline{0 \le t \le 2}$.

① 4200
② 4400
③ 4600
④ 4800
⑤ 5000

14. A particle moves along a line so that the velocity (meters per second) at time t (seconds) is $v(t) = -8 + 12t - 4t^2$. Find the distance (meters) of the particle traveled during the time period $0 \le t \le 3$.

① $\frac{22}{3}$
② $\frac{23}{3}$
③ 8
④ $\frac{25}{3}$
⑤ $\frac{26}{3}$

15. Choose every correct statement in the box below.

Ⓐ All continuous functuons have antiderivatives. Ⓑ If f is continuous on $[a,b]$, then $\frac{d}{dx}\left(\int_a^b f(x)dx\right) = f(x)$. Ⓒ If f has a discontinuity at 0, then $\int_{-1}^{1} f(x)dx$ does not exist.

① Ⓐ only
② Ⓐ and Ⓑ only
③ Ⓑ and Ⓒ only
④ Ⓐ and Ⓒ only
⑤ Ⓐ, Ⓑ, and Ⓒ

16. Choose every correct statement in the box below.

Ⓐ If oil leaks from a tank at a rate of $r(t)$ liter per miutes at time t, then $\int_0^{120} r(t)dt$ represents the amount leaked for 2 hours. Ⓑ If the units for x are meters and the units for $a(x)$ are kilograms per meter, then the units for $\int_{x_1}^{x_2} a(x)dx$ are kilograms. Ⓒ If $v(t)$ is the speed of an object moving along a straight line, then $\int_{t_1}^{t_2} v(t)dt$ is the distance of the object traveled between $[t_1, t_2]$.

① Ⓐ only
② Ⓐ and Ⓑ only
③ Ⓑ and Ⓒ only
④ Ⓐ and Ⓒ only
⑤ Ⓐ, Ⓑ, and Ⓒ

17. Choose the integral using <u>disk method</u> for the volume V of the solid obtained by rotating the region bounded by $y=x^2$, $y=4$, and $x=0$ about the y axis.

① $V=\pi\int_0^4 \sqrt{y}\,dy$

② $V=\pi\int_0^4 y\,dy$

③ $V=\pi\int_0^4 y\sqrt{y}\,dy$

④ $V=\pi\int_0^4 y^2 dy$

⑤ $V=\pi\int_0^4 y^3 dy$

18. Choose the integral using <u>disk method</u> for the volume V of the solid obtained by rotating the region bounded by $y=3-\sqrt{x}$, $y=3$, $x=1$ and $x=4$ about the x axis.

① $V=\pi\int_1^4 [3-(3-\sqrt{x})]^2 dx$

② $V=\pi\int_1^4 [3^2-(3-x)^2]dx$

③ $V=\pi\int_1^4 [3^2-(3-\sqrt{x})]dx$

④ $V=\pi\int_1^4 [3^2-(3-\sqrt{x})^2]dx$

⑤ $V=\pi\int_1^4 [3-(3-\sqrt{x})^2]dx$

19. Choose the integral using <u>shell method</u> for the volume V of the solid obtained by rotating the region bounded by $y=x^2$, $y=0$, and $x=2$ about the y axis.

① $V=2\pi\int_0^2 \sqrt{x}\,dx$

② $V=2\pi\int_0^2 x\,dx$

③ $V=2\pi\int_0^2 x\sqrt{x}\,dx$

④ $V=2\pi\int_0^2 x^2\,dx$

⑤ $V=2\pi\int_0^2 x^3\,dx$

20. Choose the integral using <u>shell method</u> for the volume V of the solid obtained by rotating the region bounded by $y=e^x$, $y=3$, and $x=0$, about the x axis.

① $V=2\pi\int_1^3 y\ln y\,dy$

② $V=2\pi\int_1^3 ye^y\,dy$

③ $V=2\pi\int_1^3 y\ln y^2\,dy$

④ $V=2\pi\int_1^3 y^2\ln y\,dy$

⑤ $V=2\pi\int_1^3 ye^{y^2}\,dy$

21. Find the work (in Joules) required to launch a 1000-kg statellite vertically to a heigt of 1000 km from planet A. You may assume that planet A's mass is 200 kg and is concentrated at its center. Take the radius of planet A to be 4000km and $G = 1.2 \times 10^3 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{kg}^2$.

① 4
② 6
③ 8
④ 10
⑤ 12

22. A tank has the shape of an inverted circular cone with a height of 8cm and a top radius of 4 cm. It is filled with oil to a height of 5cm. The oil is to be pumped out through an outlet that is 2cm above the top of the tank. Choose the correct integral for the work (centimeter-grams) required to empty the tank. (Assume the density of oil is 0.9 grams per cubic centimeters. Let the center of the top be the origin. Let one of the radius as x-axis. Let the depth from the base as y-axis.

① $\pi \int_0^5 0.9 \frac{(8-y)^2}{4}(y+2)dy$

② $\pi \int_0^5 0.9 \frac{(8-y)}{4}(y+2)dy$

③ $\pi \int_3^8 0.9 \frac{(8-y)^2}{4}(y+2)dy$

④ $\pi \int_3^8 0.9 \frac{(8-y)}{4}(y+2)dy$

⑤ $\pi \int_3^8 0.9 \frac{(8-y)^2}{4}(y+2)^2 dy$

23. The box below is the process of proving the Mean Value Theorem for Integrals by applying the Mean Value Theorem for derivatives to the function $F(x) = \int_a^x f(t)\,dt$. Choose the correct expression for

$\boxed{\text{Ⓐ}}$.

> Let $F(x) = \int_a^x f(t)\,dt$ for x in $[a,b]$.
>
> Then F is continuous on $[a,b]$ and differentiable on (a,b), so by the Mean Value Theorem there is a number c in (a,b) such that $F(b) - F(a) = F'(c)(b-a)$.
>
> But $F'(x) = f(x)$ by the Fundamental Theorem of Calculus.
>
> Therefore, $\int_a^x f(t)\,dt - 0 = \boxed{\text{Ⓐ}}$.

① $f'(c)(b-a)$

② $f(c)(b-a)$

③ $f(c)(b-a)^2$

④ $\dfrac{f(c)}{b-a}$

⑤ $\dfrac{f'(c)}{b-a}$

24. The following figure shows a hemispherical water tank with a radius of 6 meters sits on top of a 12-meter-tall tower like a dome shape. The tank is being filled by a hose connected to the bottm of the hemisphere. If a 1-horsepower pump is used to deliver water up to the tank, how long (in minutes) will it take to fill the tank completely? (Assume pi as 3, gravitational acceleration as $10\,\text{m/s}^2$, density of water as $1000\,\text{kg/m}^3$ and 1 horsepower as $750\,J/s$)

① 1360

② 1363

③ 1366

④ 1368

⑤ 1370